# 1 Number Systems

## Lesson at a Glance

1. The collection of real numbers is made up by all the rationals and irrationals.
2. Every real number is either a rational or an irrational number.
3. Every rational number can be written in the form $\frac{p}{q}$, where $p$ and $q$ are integers with $q \neq 0$.
4. No irrational number can be written in the form $\frac{p}{q}$, where $p$ and $q$ are integers with $q \neq 0$.
5. The decimal expansion of a rational number is either terminating or non-terminating repeating (recurring).
6. The decimal expansion of an irrational is non-terminating non-repeating (recurring).
7. 0, 1 and $\frac{22}{7}$ are rational numbers.
8. 2.8333...... or $2.8\overline{3}$ is rational number.
9. $\frac{22}{7}$ is not an exact value of $\pi$.
10. $\pi$ is an irrational number.
11. 0.4040040004...... is an irrational number.
12. Every integer or a fraction made up by them is a rational number.
13. Square root of every prime number is an irrational number.

14. There are infinitely many rational numbers between any two distinct rational numbers.
15. All the rational and irrational numbers lie on the number line.
16. Every real number is represented by a unique point on the number line.
17. The sum or difference of a rational and an irrational number is an irrational number.
18. The product or quotient of a non-zero rational and an irrational number is an irrational number.

## TEXTBOOK QUESTIONS SOLVED

### Exercise 1.1 *(Page – 5)*

1. *Is zero a rational number? Can you write it in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$?*

**Sol.** Yes, zero is a rational number as $0 = \frac{0}{1}$ or $\frac{0}{2}$ or $\frac{0}{-1}$.

This is in the form $\frac{p}{q}$, $q \neq 0$.

2. *Find six rational numbers between 3 and 4.*

**Sol.** For six rational numbers between 3 and 4,

$$3 = \frac{21}{7} \text{ and } 4 = \frac{28}{7}.$$

Six rational numbers between 3 and 4 are $\frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7}, \frac{27}{7}$ or another set is 3.1, 3.2, 3.3, 3.4, 3.5, 3.6. There can be other set of rational numbers also.

3. *Find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.*

**Sol.** We have $\frac{3}{5} = \frac{18}{30}$ and $\frac{4}{5} = \frac{24}{30}$.

Five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$ are $\frac{19}{30}, \frac{20}{30},$

$\frac{21}{30}, \frac{22}{30}, \frac{23}{30}$. There can be other set of rational numbers also.

4. *State whether the following statements are true or false. Give reasons for your answers.*
   (*i*) *Every natural number is a whole number.*
   (*ii*) *Every integer is a whole number.*
   (*iii*) *Every rational number is a whole number.*

**Sol.** (*i*) True, as the set of whole numbers contains all the natural numbers.
(*ii*) False, as negative integer, *e.g.*, –2 is not a whole number.
(*iii*) False, as $\frac{2}{3}$ is a rational number but not a whole number.



## Exercise 1.2 *(Page – 8)*

1. *State whether the following statements are true or false. Justify your answers.*
   (*i*) *Every irrational number is a real number.*
   (*ii*) *Every point on the number line is of the form $\sqrt{m}$, where m is a natural number.*
   (*iii*) *Every real number is an irrational number.*

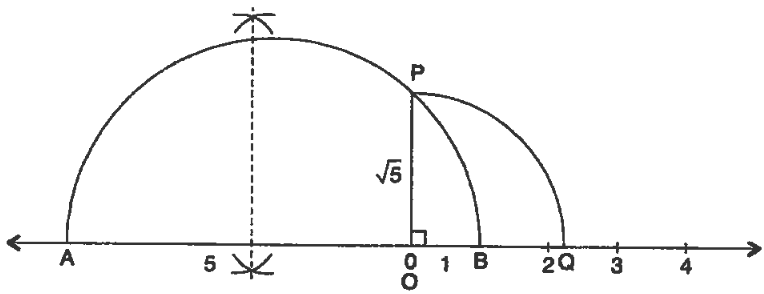
**Sol.** (*i*) True, as real numbers consist of rational and irrational numbers.
(*ii*) False, as $\frac{3}{2}$ on the number line cannot be a square root of a natural number.
Also, a negative number cannot be a square root of natural number, as $\sqrt{m}$ represents a positive value.
(*iii*) False, as 2 is a real number but not an irrational number.

2. *Are the square roots of all positive integers irrational? If not, give an example of the square root of a number that is a rational number.*

**Sol.** No. For example, $\sqrt{9} = 3$, $\sqrt{16} = 4$, etc., 3, 4 are rational numbers.

3. *Show how $\sqrt{5}$ can be represented on the number line.*

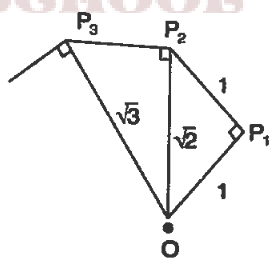
**Sol.** For $\sqrt{5}$, we have 5 = 5 × 1. On a number line, take O at position 0 and OA = 5 and OB = 1. With AB as diameter draw a semicircle. Draw OP ⊥ AB meeting semicircle at P. With O as centre and OP as radius an arc is drawn meeting the number line at Q.




Then, OQ = OP = $\sqrt{5}$ and Q represents $\sqrt{5}$ on the number line.



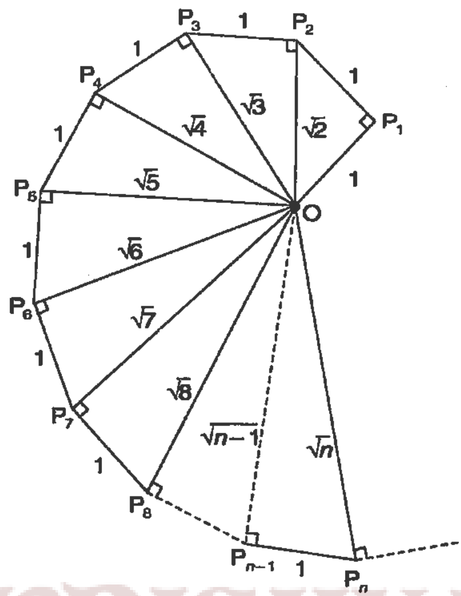
4. ***Classroom activity (Constructing the 'square root spiral'):*** *Take a large sheet of paper and construct the 'square root spiral' in the following fashion. Start with a point O and draw a line segment $OP_1$ of unit length. Draw a line segment $P_1P_2$ perpendicular to $OP_1$ of unit length (see Figure).*




**Constructing square root spiral**

*Now, draw a line segment $P_2P_3$ perpendicular to $OP_2$ of unit length. Then, draw a line segment $P_3P_4$ perpendicular to $OP_3$ of unit length. Continuing in this manner, you can get the line segment $P_{n-1}P_n$ by drawing a line segment of unit length perpendicular to $OP_{n-1}$. In this manner, you will have created the points $P_2$, $P_3$, ......., $P_n$, ......., and joined them to create a beautiful spiral depicting $\sqrt{2}$, $\sqrt{3}$, $\sqrt{4}$, ...... .*

**Sol.**




**Square root spiral**



## Exercise 1.3 *(Page – 14)*

**1.** *Write the following in decimal form and say what kind of decimal expansion each has:*

(*i*) $\frac{36}{100}$ (*ii*) $\frac{1}{11}$ (*iii*) $4\frac{1}{8}$

(*iv*) $\frac{3}{13}$ (*v*) $\frac{2}{11}$ (*vi*) $\frac{329}{400}$.

**Sol.** (*i*) $\frac{36}{100}$ = 0.36, terminating decimal expansion.

(*ii*) $\frac{1}{11} = 0.090909..... = 0.\overline{09}$, non-terminating repeating decimal expansion.

(*iii*) $4\frac{1}{8} = \frac{33}{8} = 4.125$, terminating decimal expansion.

(*iv*) $\frac{3}{13} = 0.230769230769..... = 0.\overline{230769}$, non-terminating repeating decimal expansion.

(v) $\frac{2}{11} = 0.181818 \ldots = 0.\overline{18}$, non-terminating repeating decimal expansion.

(vi) $\frac{329}{400} = 0.8225$, terminating decimal expansion.

**2.** *You know that $\frac{1}{7} = 0.\overline{142857}$. Can you predict what the decimal expansions of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$ are without actually doing the long division? If so, how?*

[**Hint:** *Study the remainders while finding the value of $\frac{1}{7}$ carefully.*]

**Sol.** Yes, we can predict the required decimal expansions.



We are given, $\frac{1}{7} = 0.\overline{142857}$

On dividing 1 by 7, we find that the remainders repeat after six divisions, therefore, the quotient has a repeating block of six digits in the decimal expansion of $\frac{1}{7}$. So, to obtain decimal expansions of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}$ and $\frac{6}{7}$; we multiply 142857 by 2, 3, 4, 5 and 6 respectively, to get the integral part and in the decimal part, we take block of six repeating digits in each case. Hence, we get

$$\frac{2}{7} = 2 \times \frac{1}{7} = 0.\overline{285714}$$

$$\frac{3}{7} = 3 \times \frac{1}{7} = 0.\overline{428571}$$

$$\frac{4}{7} = 4 \times \frac{1}{7} = 0.\overline{571428}$$

$$\frac{5}{7} = 5 \times \frac{1}{7} = 0.\overline{714285}$$

and $\frac{6}{7} = 6 \times \frac{1}{7} = 0.\overline{857142}$.

3. *Express the following in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$:*
*(i) $0.\overline{6}$* *(ii) $0.4\overline{7}$* *(iii) $0.\overline{001}$.*

**Sol.** (*i*) Let $x = 0.\overline{6}$

or $x = 0.666...$ ...(*i*)

$10x = 6.666...$ ...(*ii*) [On multiplying (*i*) by 10]

$\Rightarrow 9x = 6$ [Subtracting (*i*) from (*ii*)]

$\therefore x = \frac{2}{3}$.

(*ii*) Let $x = 0.4\overline{7}$

or $x = 0.4777...$ ...(*i*)

Multiplying (*i*) by 10, we get

$10x = 4.777...$ ...(*ii*)

Again multiplying (*ii*) by 10, we get

$100x = 47.777...$ ...(*iii*)

Subtracting equation (*ii*) from equation (*iii*), we get

$90x = 43$

$\therefore x = \frac{43}{90}$.

(*iii*) Let $x = 0.\overline{001}$

or $x = 0.001001...$ ...(*i*)

$\Rightarrow 1000x = 1.001001...$ ...(*ii*)

[On multiplying (*i*) by 1000]

$\Rightarrow 999x = 1$ [On subtracting (*i*) from (*ii*)]

$\therefore x = \frac{1}{999}$.

4. *Express 0.99999... in the form $\frac{p}{q}$. Are you surprised by your answer? With your teacher and classmates, discuss why the answer makes sense.*

**Sol.** Let $x = 0.99999...$ ...(*i*)

$\Rightarrow 10x = 9.99999...$ ...(*ii*) [On multiplying (*i*) by 10]

$\Rightarrow 9x = 9$ [On subtracting (*i*) from (*ii*)]

$\therefore x = 1$.

Yes, we are surprised by our answer.

**5.** ***What can the maximum number of digits be in the repeating block of digits in the decimal expansion of $\frac{1}{17}$? Perform the division to check your answer.***

**Sol.**

```
          0.0588235294117647...
     17) 1.0000000000000000
          00
          100
           85
           150
           136
            140
            136
              40
              34
               60
               51
                90
                85
                 50
                 34
                 160
                 153
                   70
                   68
                    20
                    17
                     30
                     17
                     130
                     119
                      110
                      102
                        80
                        68
                        120
                        119
                          1  ←—— Repeating
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Thus, $\frac{1}{17} = 0.\overline{0588235294117647}$

Hence, the required number of digits in the repeating block is 16.

**6.** *Look at several examples of rational numbers in the form $\frac{p}{q}$ $(q \neq 0)$, where p and q are integers with no common factors other than 1 and having terminating decimal representations (expansions). Can you guess what property q must satisfy?*

**Sol.** Examples are $\frac{3}{4}, \frac{4}{5}, \frac{7}{8}, \frac{9}{10}$, etc.

$q$ have only powers of 2 or powers of 5 or both.

**7.** *Write three numbers whose decimal expansions are non-terminating non-recurring.*

**Sol.** (*i*) 2.010110111011110111111....... (*ii*) 0.03003000300003.....
(*iii*) 4.12112111211112.......



**8.** *Find three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$.*

**Sol.** Given: $\frac{5}{7} = 0.\overline{714285}$ and $\frac{9}{11} = 0.\overline{81}$

We can have irrational numbers as 0.72072007200072.......; 0.801001800018.......; 0.74301010010001......;

**9.** *Classify the following numbers as rational or irrational:*

(*i*) $\sqrt{23}$ (*ii*) $\sqrt{225}$ (*iii*) *0.3796*

(*iv*) *7.478478.....* (*v*) *1.101001000100001....*

**Sol.** (*i*) $\sqrt{23}$. As it is square root of a prime number, so, irrational number.

(*ii*) $\sqrt{225} = 15$, rational number.

(*iii*) 0.3796, terminating decimal, so rational number.

(*iv*) 7.478478..... $= 7.\overline{478}$, non-terminating repeating (recurring), so rational number.

(v) 1.101001000100001....... non-terminating non-repeating, so irrational number.

## Exercise 1.4 *(Page – 18)*

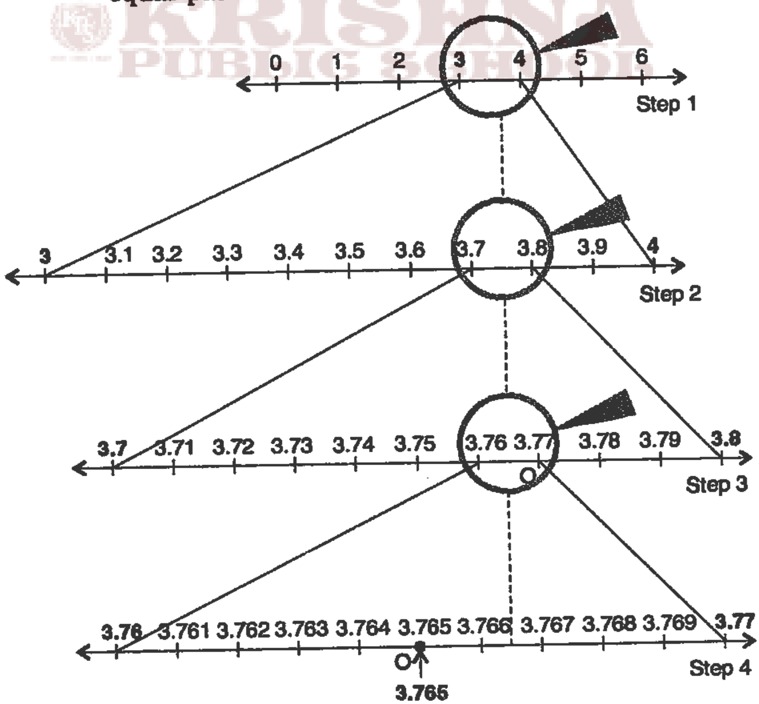
1. *Visualise 3.765 on the number line, using successive magnification.*

**Sol.** (i) We notice the number 3.7 lies between 3 and 4. So, first we locate numbers 3 and 4 on number line and divide the portion into ten equal parts and locate 3.7 and 3.8. [Refer step 2]

(ii) Further 3.76 lies between 3.7 and 3.8. So, we magnify 3.7 and 3.8 and divide the portion into ten equal parts and locate 3.76 and 3.77. [Refer step 3]

(iii) Further 3.765 lies between 3.76 and 3.77. So, we magnify 3.76 and 3.77 and divide the portion into ten equal parts and locate 3.765. [Refer step 4]






Point O in step 4 represents the number 3.765 on the number line.

2. *Visualise $4.\overline{26}$ on the number line, up to 4 decimal places.*

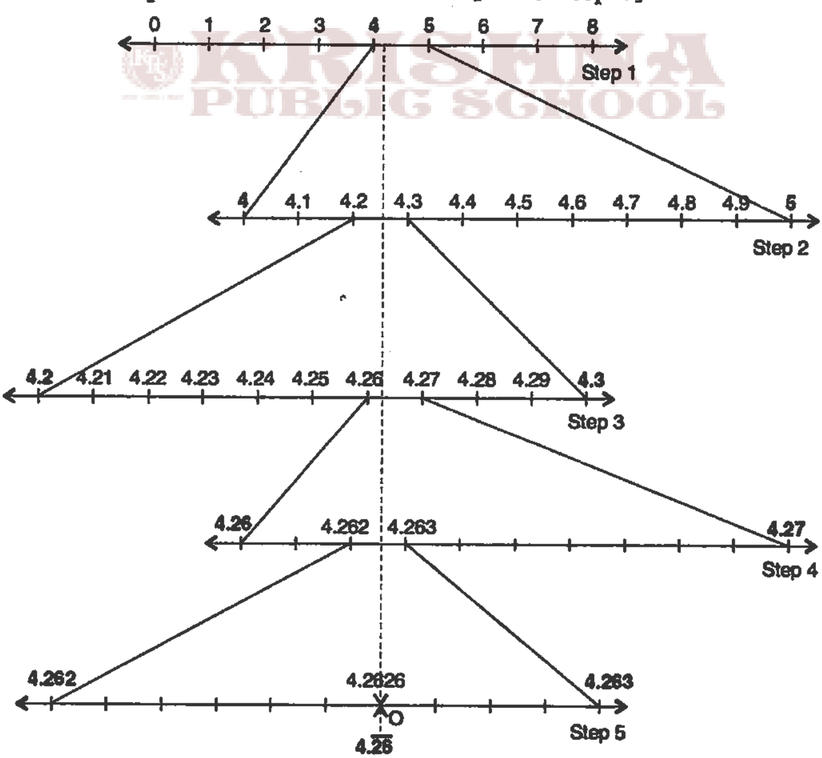
**Sol.** $4.\overline{26}$ = 4.2626262626..........

(*i*) Visualise 4 and 5 as $4.\overline{26}$ lies between 4 and 5 and divide portion in ten equal parts and locate 4.2. [Refer step 2]

(*ii*) Visualise 4.2 and 4.3 as 4.26 lies between 4.2 and 4.3 and divide portion in ten equal parts and locate 4.26. [Refer step 3]

(*iii*) Visualise 4.26 and 4.27 as 4.262 lies between 4.26 and 4.27 and divide portion in ten equal parts and locate 4.262. [Refer step 4]

(*iv*) Visualise 4.262 and 4.263 as 4.2626 lies between 4.262 and 4.263 and divide portion in ten equal parts and locate 4.2626. [Refer step 5]



Point O in step 5 represents the number $4.\overline{26}$ on the number line.

## Exercise 1.5 *(Page – 24)*

**1.** *Classify the following numbers as rational or irrational:*

(*i*) $2 - \sqrt{5}$ (*ii*) $(3 + \sqrt{23}) - \sqrt{23}$

(*iii*) $\dfrac{2\sqrt{7}}{7\sqrt{7}}$ (*iv*) $\dfrac{1}{\sqrt{2}}$ (*v*) $2\pi$.

**Sol.** (*i*) $2 - \sqrt{5}$ is an irrational number, as difference of a rational and an irrational number is irrational.

(*ii*) $(3 + \sqrt{23}) - \sqrt{23} = 3 + \sqrt{23} - \sqrt{23} = 3$, is a rational number.

(*iii*) $\dfrac{2\sqrt{7}}{7\sqrt{7}} = \dfrac{2}{7}$, is a rational number.



(*iv*) $\dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$ is an irrational number, as divisors of an irrational number by a non-zero rational number is irrational.

(*v*) $2\pi$, irrational number, as $\pi$ is an irrational number and multiplication of a rational and an irrational number is irrational.

**2.** *Simplify each of the following expressions:*

(*i*) $(3 + \sqrt{3})(2 + \sqrt{2})$ (*ii*) $(3 + \sqrt{3})(3 - \sqrt{3})$

(*iii*) $(\sqrt{5} + \sqrt{2})^2$ (*iv*) $(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2})$

**Sol.** (*i*) $(3 + \sqrt{3})(2 + \sqrt{2}) = 6 + 3\sqrt{2} + 2\sqrt{3} + \sqrt{6}$

(*ii*) $(3 + \sqrt{3})(3 - \sqrt{3}) = (3)^2 - (\sqrt{3})^2 = 9 - 3 = 6.$

(*iii*) $(\sqrt{5} + \sqrt{2})^2 = (\sqrt{5})^2 + 2.\sqrt{5}.\sqrt{2} + (\sqrt{2})^2$

$= 5 + 2\sqrt{10} + 2 = 7 + 2\sqrt{10}.$

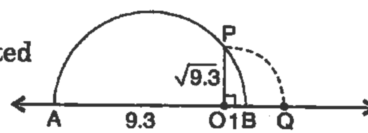
(*iv*) $(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2}) = (\sqrt{5})^2 - (\sqrt{2})^2 = 5 - 2 = 3.$

3. *Recall, $\pi$ is defined as the ratio of the circumference (say c) of a circle to its diameter (say d). That is, $\pi = \frac{c}{d}$. This seems to contradict the fact that $\pi$ is irrational. How will you resolve this contradiction?*

**Sol.** On measuring $c$ with any device, we get only approximate measurement. Therefore, $\pi$ is an irrational.

4. *Represent $\sqrt{9.3}$ on the number line.*

**Sol.** $\sqrt{9.3} = \sqrt{9.3 \times 1}$




Let position 0 be represented by O on the number line.

Let OA = 9.3 and OB = 1.

With AB as diameter draw a semicircle. Draw OP perpendicular to AB, meeting the semicircle at P. Then OP = $\sqrt{9.3}$. With O as centre and OP as radius draw an arc to meet the number line at Q on the positive side. Then, OQ = $\sqrt{9.3}$ and the point Q thus obtained represents $\sqrt{9.3}$.

5. *Rationalise the denominators of the following:*

(i) $\frac{1}{\sqrt{7}}$ (ii) $\frac{1}{\sqrt{7}-\sqrt{6}}$

(iii) $\frac{1}{\sqrt{5}+\sqrt{2}}$ (iv) $\frac{1}{\sqrt{7}-2}$.

**Sol.** (i) $\frac{1}{\sqrt{7}} = \frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{7}}{7}$.

(ii) $\frac{1}{\sqrt{7}-\sqrt{6}} = \frac{\sqrt{7}+\sqrt{6}}{(\sqrt{7}-\sqrt{6})(\sqrt{7}+\sqrt{6})} = \frac{\sqrt{7}+\sqrt{6}}{7-6} = \sqrt{7} + \sqrt{6}$.

(iii) $\frac{1}{\sqrt{5}+\sqrt{2}} = \frac{\sqrt{5}-\sqrt{2}}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})} = \frac{\sqrt{5}-\sqrt{2}}{5-2} = \frac{\sqrt{5}-\sqrt{2}}{3}$.

(iv) $\frac{1}{\sqrt{7}-2} = \frac{\sqrt{7}+2}{(\sqrt{7}-2)(\sqrt{7}+2)} = \frac{\sqrt{7}+2}{7-4} = \frac{\sqrt{7}+2}{3}$.

## Exercise 1.6 *(Page – 26)*

**1.** *Find:* (i) $64^{1/2}$ (ii) $32^{1/5}$ (iii) $125^{1/3}$.

**Sol.** (i) $64^{1/2} = (8^2)^{1/2} = (8)^{2 \times 1/2} = 8.$

(ii) $32^{1/5} = (2^5)^{1/5} = (2)^{5 \times 1/5} = 2.$

(iii) $125^{1/3} = (5^3)^{1/3} = (5)^{3 \times 1/3} = 5.$

**2.** *Find:* (i) $9^{3/2}$ (ii) $32^{2/5}$ (iii) $16^{3/4}$ (iv) $125^{-1/3}$.

**Sol.** (i) $9^{3/2} = (3^2)^{3/2} = (3)^{2 \times 3/2} = (3)^3 = 27.$

(ii) $32^{2/5} = (2^5)^{2/5} = 2^{5 \times 2/5} = (2)^2 = 4.$

(iii) $16^{3/4} = (2^4)^{3/4} = (2)^{4 \times 3/4} = (2)^3 = 8.$

(iv) $(125)^{-1/3} = (5^3)^{-1/3} = (5)^{3 \times (-1/3)} = (5)^{-1} = \frac{1}{5}.$

**3.** *Simplify:* (i) $2^{2/3} \cdot 2^{1/5}$ (ii) $\left(\frac{1}{3^3}\right)^7$ (iii) $\frac{11^{1/2}}{11^{1/4}}$ (iv) $7^{1/2} \cdot 8^{1/2}$.



**Sol.** (i) $2^{2/3} \cdot 2^{1/5} = 2^{2/3 + 1/5} = 2^{13/15}.$

(ii) $\left(\frac{1}{3^3}\right)^7 = \frac{1}{(3^3)^7} = \frac{1}{3^{3 \times 7}} = \frac{1}{3^{21}} = 3^{-21}.$

(iii) $\frac{11^{1/2}}{11^{1/4}} = 11^{1/2 - 1/4} = 11^{1/4}.$

(iv) $7^{1/2} \cdot 8^{1/2} = (7 \cdot 8)^{1/2} = 56^{1/2}.$